7.1 Laws of Force

SECTION EXPECTATIONS

- Define and describe the concepts related to electric, gravitational, and magnetic fields.
- Analyze and compare Coulomb's law and Newton's law of universal gravitation.
- Apply quantitatively Coulomb's law and Newton's law of universal gravitation.
- Collect, analyze, and interpret data from experiments on charged particles.

KEY TERMS

- inverse square law
- electrostatic force
- torsion balance
- Coulomb's law
- Coulomb's constant

In science courses over the past several years, you have gained experience in applying the laws of motion of Sir Isaac Newton (1642–1727) and analyzing the motion of many types of objects. The two forces that you encounter most frequently are the forces of gravity and friction. In many cases, you have also dealt with an applied force, in which one object or person exerted a force on another. In this unit, you will focus on the nature of the forces themselves.

Gravity and the Inverse Square Law

Several astronomers and other scientists before Newton developed the concept that the force of gravity obeyed an **inverse square law**. In other words, the magnitude of the force of gravity between two masses is proportional to the inverse of the square of the distance separating their centres: $F \propto \frac{1}{r^2}$. It was Newton, though, who verified the relationship.

Figure 7.1 The centripetal force that keeps the Moon in its orbit is the gravitational force between Earth and the Moon.

Newton reasoned that, since the Moon is revolving around Earth with nearly circular motion, the gravitational force between Earth and the Moon must be providing the centripetal force. His reasoning was similar to the following.

- Write the equation for centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

- Write the equation for speed.

$$v = \frac{\Delta d}{\Delta t}$$

- The Moon travels the circumference of an orbit in one period. Therefore, its speed is

$$\Delta d = 2\pi r = 2\pi(3.84 \times 10^8 \text{ m}) = 2.41 \times 10^9 \text{ m}$$

$$T = 2.36 \times 10^6 \text{ s}$$

$$v = \frac{2\pi r}{T} = \frac{2.41 \times 10^9 \text{ m}}{2.36 \times 10^6 \text{ s}} = 1.02 \times 10^3 \, \frac{\text{m}}{\text{s}}$$

- The centripetal acceleration of the Moon is therefore

$$a_c = \frac{v^2}{r} = \frac{\left(1.02 \times 10^3 \ \frac{\text{m}}{\text{s}}\right)^2}{3.84 \times 10^8 \text{ m}} = 2.71 \times 10^{-3} \ \frac{\text{m}}{\text{s}^2}$$

- If the force of gravity decreases with the square of the distance between the centre of Earth and the centre of the Moon, then the acceleration due to gravity should also decrease. Write the inverse square relationships and divide the first by the second.

$$a_{g(\text{Moon})} \propto \frac{1}{r^2_{\text{E-Moon}}} \qquad a_{g(\text{Moon})} = \frac{GM_E}{r^2_{\text{E-Moon}}}$$

$$g \propto \frac{1}{r^2_E} \qquad g = \frac{GM_E}{r^2_E}$$

$$\frac{a_{g(\text{Moon})}}{g} = \frac{\frac{1}{r^2_{\text{E-Moon}}}}{\frac{1}{r^2_E}} = \frac{r^2_E}{r^2_{\text{E-Moon}}}$$

- In the ratio above, solve for the acceleration due to gravity at the location of the Moon. Insert the value of g and the distances.

$$a_{c(\text{Moon})} = \frac{g r^2_E}{r^2_{\text{E-Moon}}}$$

$$a_{c(\text{Moon})} = \frac{\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(6.38 \times 10^6 \text{ m})^2}{(3.84 \times 10^8 \text{ m})^2}$$

$$a_{c(\text{Moon})} = 2.71 \times 10^{-3} \ \frac{\text{m}}{\text{s}^2}$$

The values of acceleration due to gravity that were calculated in two completely different ways are in full agreement. The centripetal acceleration of the Moon in orbit is exactly what you would expect it to be if that acceleration was provided by the force of gravity and if the force of gravity obeyed an inverse square law.

The force of gravity exerts its influence over very long distances and is the same in all directions, suggesting that the influence extends outward like a spherical surface. The equation relating the surface area of a sphere to its radius is $A = 4\pi r^2$, or the area of a sphere increases as the square of the radius. You can relate the influence of the force of gravity with a portion of a spherical surface, A, at a distance r, as shown in Figure 7.2. When the distance doubles to $2r$, the area increases by 2^2, or four. When the distance increases to $4r$, the area of the sphere increases by 4^2, or 16. The influence of the force of gravity appears to be spreading out over the surface area of a sphere. How does this property of the force of gravity compare to the electromagnetic force?

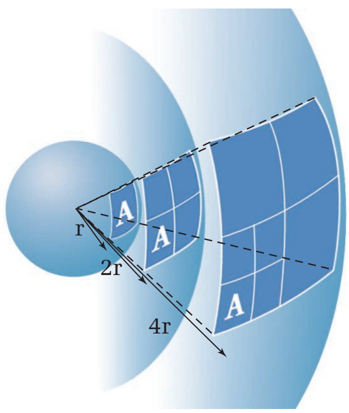

Figure 7.2 The intensity of physical phenomena that obey inverse square laws can be compared to the spreading out of the surface of a sphere.

COURSE CHALLENGE

Contact versus Non-Contact

Action at a distance — something that might have seemed magical to you as a child — lies at the heart of several cutting-edge technologies. Refer to page 604 of this textbook for suggestions about non-contact interactions for your Course Challenge project.